Screening Contracts for Product and Process Development: A Principal-Agent Model

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Abstract

Consider a product and process development project involving two parties: a “buyer” who expends effort to develop product specifications, and a supplier who expends effort to develop process specifications and produce the “product.” The supplier’s total production cost depends on the buyer’s product specification effort, the supplier’s process specification effort, and the supplier’s production capability. The supplier’s capability is hidden from the buyer, and the buyer’s problem is to determine a contract mechanism to minimize her total expected cost, which includes the supplier’s production cost and a contract fee. We develop a principal-agent model to analyze a contract in which the buyer specifies a single product specification and price, a product-specification screening contract, and a product-process screening contract. In the screening contracts, the buyer proposes a menu of “options” and prices, and the supplier reveals his capability by the choice he makes. Theoretical and numerical results demonstrate that the nature of the two screening contracts depends on whether the buyer’s production effort and the supplier’s capability are complements or substitutes. Furthermore, screening contracts reduce the buyer’s total expected cost compared to the single product-specification contract in the case of complements but not in the case of substitutes. This result provides theoretical justification for the wide use of product development contracts in which product specifications are not adapted to supplier capabilities (i.e. single product specification contracts), and identifies conditions under which such contracts will be inefficient.

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1 Introduction

“Times have changed since Henry Ford made the River Rouge complex in Dearborn, Michigan, into the ultimate in vertical integration, with iron ore going in at one end and shiny model A’s coming out the other. Now vertical dis-integration is the order of the day - in autos, in handheld computers, in pharmaceuticals, in ink-jet printers, in health care, in cameras....” Forbes, April 30, 2001

“Good-bye mergers and acquisitions. In a global market tied together by the internet, corporate partnerships and alliances are proving a more productive way to keep companies growing.” Forbes Best of the Web, May 21, 2001

Outsourcing and partnering are changing the nature of companies’ organizational charts and providing a new source for competitive advantage: the ability to form and manage effective buyer-supplier relationships. In the automobile industry, for example, Chrysler outsources over 80% of the parts it assembles into cars; Ford, over 65%, and GM, over 55%. Cisco System partners provide final assembly for almost half of its switches and routers. Nearly 80% of Kodak’s reloadable cameras are sourced in Asia, while nearly all of HP’s printers are made by someone else. See Forbes (2001).

Moreover, outsourcing is not limited to the production of parts, final assembly, or the entire production process. Increasingly, firms are outsourcing parts of the product life-cycle for their products and services. In pharmaceutical manufacture, for example, Pfizer outsources pilot production, and scale-up and manufacturing. In pharmaceutical research, Merck performs much of its own R&D, but outsources testing and the evaluation of results. See Quinn (2000).

Outsourcing is increasingly common in services, too. In financial services, for example, PNC and First Union develop the conceptual specifications for services, but outsource their sales and marketing, transactions processing and customer-relationship management to MBNA. Citigroup recently outsourced the development of its payment and settlement capabilities to Oracle. Further, Quinn (2000) recently described competitive forces that are
leading to the outsourcing of innovation - e.g., basic research, early-stage research, advanced development, business processes, and new-product introductions - in pharmaceuticals, consumer electronics, and telecommunications.

Our research addresses the topic of assessing supplier capability and managing an effective relationship through contracting. In particular we examine contracting between a “buyer” and a single “supplier” regarding “product development” and “production” of a single “product”. However, since our model is stylized, it can be applied to the outsourcing of any contiguous element in the production of products (e.g., design, component production, assembly) or the provision of services (e.g., brand management, sales and marketing).

Model Overview and Summary of Main Results. We visualize the development and production process as three sequential steps: In step 0, the buyer has already developed the initial design, or concept specification, for the product. Step 0 can be viewed as the buyer having a “black-box” design (Karlson, et al., 1998). In step 1, the buyer develops the product specification. In particular we focus on the effort the buyer expends to “fine tune” the design in order to assure its quality, functionality, design for manufacturability, etc. Increasing buyer effort can be viewed as providing increasingly detailed specifications with respect to the “black-box”, its materials, and/or processing. In step 2, production, the supplier develops the production process and provides the product or service as specified in the contract. Our analysis focuses on steps 1 and 2.

We examine this model from a principal-agent perspective, wherein the buyer (she) is the principal who uses the supplier (he) as an agent to produce the product according to the product specifications. Unlike typical principal-agent models, however, the buyer is able to undertake effort (in step 1) that makes the supplier’s task easier and reduces total cost. Our analysis assumes that a fixed level of resources are available to support this effort. These resources may be viewed as part of the overhead or indirect expense associated with outsourcing. The buyer does not know the capability of the supplier and she may not be
able to observe the effort of the supplier.

The buyer’s objective is to minimize the total expected cost of production, which includes the cost of all materials, plus a contract fee, or price, to the supplier. The buyer minimizes total expected cost by: (1) selecting her level of effort in product development; and (2) specifying the terms of the supplier contract (product specification, possibly process specification, and price). The supplier is in charge of production, and his total production cost is a function of the buyer’s product specification, and his own production effort and production capability. The supplier’s production effort includes among other things his choice and implementation of a particular production process. Therefore, we will use process specification to denote the supplier’s production effort. The supplier’s capability is assumed unobservable by the buyer but observable by the supplier. We consider a very general three-feature cost function: a) the total production cost decreases as the buyer expends more effort on the product specifications; b) it also decreases with the supplier’s capability; and c) for each product specification, there is a unique cost-minimizing process specification which depends on the supplier’s capability. The model also make the implicit assumptions that the quality and quantity of the “product” is fixed, that the buyer-supplier interaction is restricted to a single period (e.g., product life-cycle), and that the buyer incurs no explicit cost for his effort.

The purpose of our analysis is to explore contract mechanisms that alleviate incentive tensions between the two parties and attain outcomes that best utilize the supplier’s capability and minimize the buyer’s total cost. In order to pursue this objective, we analyze three distinct contract mechanisms and for each, we identify the contract parameters that minimize the buyer’s total expected cost and ensure the participation of the supplier (i.e., the supplier’s profit must exceed a reservation level). The simplest mechanism we consider is the single product-specification contract, where the buyer specifies a single product specification and proposes a fixed price. Our analysis provides the following insights about the
nature of this contract. First, because this mechanism does not impose any conditions on the supplier-chosen process specification, the supplier will choose the specification that minimizes his total cost. Second, because the buyer wants the supplier to accept this contract, she offers a price that is high enough to secure the participation of the least capable supplier. Furthermore, she uses all her resources to provide the most detailed product specification because that will result into the lowest price. As a result, the optimal product and process specification that emerge for this contract coincide with first-best. However, all the surplus generated from cost minimization is retained by the supplier. Supplier surplus is defined as the difference between the supplier’s production cost and the payment he receives from the buyer.

The second contracting mechanism is a product-process screening contract. Here it is assumed that the buyer can write a contract that specifies both the product and the process specification (i.e. the production effort of the supplier). Furthermore, to exploit the supplier’s capability, the buyer proposes a menu of product-process specification pairs (i.e. a so-called screening contract) and prices, whereby each pair in the menu is tailored to a supplier of certain capability. The supplier will reveal his capability by his choice of a product-process pair. Truthful revelation of the supplier’s capability is achieved by offering a price that includes an information rent which is increasing in the supplier’s capability. Our analysis shows that the nature of the optimal contract depends on the interaction between the level of detail in the product specification and the supplier’s capability. Specifically, we consider two cases: One where product specification and supplier’s capability are complements, and a second one where they are substitutes. Mathematically, the complements (substitutes) case assumes that more detail in the product specification (i.e. buyer effort) not only lowers the total cost, but it also increases (decreases) the magnitude of the supplier’s marginal cost reduction capability. In the complements case, the optimal contract menu includes variations both in product and in process specification, and, under the optimal
contract, more capable suppliers choose more detailed product specifications, adopt a more challenging process, and, in exchange receive, a price premium. By contrast, in the substitutes case, the contract specifies a single product specification and a variety of processes. The more capable supplier adopts a more challenging process as before.

Since it may not be possible to write enforceable product-process screening contracts, we also analyze a product-screening contract, that specifies a menu of product specifications and leaves the choice of process specification up to the supplier. Our analysis shows that, compared to the optimal product-process screening contract, the product-screening contract leaves the supplier with a bigger share of the surplus. We also demonstrate that under certain circumstances it is optimal to “bunch” all suppliers together and offer a single product specification.

The remainder of this paper is organized as follows. Section 2 describes the model and section 3 presents the analysis and main results. Section 4 outlines a numerical example that illustrates the main results from section 3. Section 5 provides managerial insights based on our analysis and corresponding observations with respect to practice. Section 6 provides the customary literature review, and the concluding remarks are given in section 7. All the proofs are given in the Appendix.

2 The Principal-Agent Model

The model incorporates two decisions: Product specification $x_1$, selected by the buyer, and process specification $x_2$, selected by the supplier. The buyer has limited resources for product specification; hence

$$0 \leq x_1 \leq R. \quad (1)$$

The total production cost $V(x_1, x_2, \phi)$ is a function of the two decision variables and of the supplier’s capability $\phi \in [\underline{\phi}, \overline{\phi}]$. The set $[\underline{\phi}, \overline{\phi}]$ will be denoted by $\Phi$. The supplier’s capability
reflects its technological and management expertise. Both parties are risk-neutral.

For the information structure, we assume that both parties know the cost function but the supplier alone knows his capability. The buyer’s uncertainty about the supplier’s capability is reflected in his prior \( \pi(\phi) \). The effect of information asymmetry is aggravated because there are limitations on the type of contracts that one can enforce. Under the least-limited contract, both product and process specification are contractible. That is, they can be specified unambiguously in a contract and deviations from the contracted upon options can be penalized in court. In a more limited (and realistic) contract, only product specification is contractible.

Before we proceed with the analysis, it is convenient to introduce the following assumptions about the cost function \( V(x_1, x_2, \phi) \) and prior density \( \pi(\phi) \):

A1 \( V(x_1, x_2, \phi) \) is strictly decreasing in \( x_1 \) and in \( \phi \), and for each \( x_1 \) and \( \phi \) there exists a finite and unique \( x_2 \) that minimizes \( V(x_1, x_2, \phi) \). Moreover, \( V(x_1, x_2, \phi) \) is jointly convex in \((x_1, x_2)\).

A2 \( \left| \frac{\partial V(x_1, x_2, \phi)}{\partial x_1} \right| \leq K_0, \left| \frac{\partial V(x_1, x_2, \phi)}{\partial x_2} \right| \leq K_0. \)

A3a \( \frac{\partial^2 V(x_1, x_2, \phi)}{\partial x_1 \partial \phi} < 0 \)

A3b \( \frac{\partial^2 V(x_1, x_2, \phi)}{\partial x_2 \partial \phi} > 0 \)

A3c \( \frac{\partial^2 V(x_1, x_2, \phi)}{\partial x_2 \partial \phi} < 0 \)

A4 (monotone hazard rate) \( \frac{d}{d \phi} \left( \frac{1-P(\phi)}{\pi(\phi)} \right) \leq 0 \) with \( \lim_{\phi \uparrow \bar{\phi}} \frac{1-P(\phi)}{\pi(\phi)} = +\infty \) and \( \lim_{\phi \downarrow \phi_1} \frac{1-P(\phi)}{\pi(\phi)} = 0 \); for brevity of notation define \( \eta(\phi) = \frac{1-P(\phi)}{\pi(\phi)} \).

A5 \( \frac{\partial^3 V(x_1, x_2, \phi)}{\partial x_1 \partial \phi^2} > 0, \frac{\partial^3 V(x_1, x_2, \phi)}{\partial x_2 \partial \phi^2} > 0. \)

A6 \( V_\phi(x_1, x_2, \phi) \) is jointly concave in \((x_1, x_2)\) for each \( \phi \).
$V_{x_1 x_2}(x_1, x_2, \phi) - \eta(\phi) V_{x_1 x_2}(x_1, x_2, \phi) < 0$ and $|V_{x_1 x_2}(x_1, x_2, \phi) - \eta(\phi) V_{x_1 x_2}(x_1, x_2, \phi)| \leq \min\{V_{x_1 x_1}(x_1, x_2, \phi) - \eta(\phi) V_{x_1 x_1}(x_1, x_2, \phi), V_{x_2 x_2}(x_1, x_2, \phi) - \eta(\phi) V_{x_2 x_2}(x_1, x_2, \phi)\}$.

These assumptions can be divided into two broad categories: Assumptions A1, A3, and A4 have a clear managerial interpretation. By contrast, assumptions A2, A5, A6, and A7 make the analysis tractable (in the sense that local optimality conditions are necessary and sufficient for global optimality) but do not admit a meaningful interpretation.

Assumptions A3a and A3b capture the interaction between product specifications and supplier capability. Specifically, A3a implies that the supplier’s capability and buyer’s production effort are complements: more buyer effort (i.e., detail) in the product specification not only lowers total cost, but it also increases the magnitude of the supplier’s marginal cost reduction capability (i.e. it decreases $\frac{\partial V(x_1, x_2, \phi)}{\partial \phi}$). In other words, when the supplier’s capability and buyer’s product specification are complements, more detail in the buyer’s product specification increases the marginal capability of the supplier. On the other hand, A3b states that supplier capability and buyer production effort are substitutes: more buyer effort in the product specification decreases the magnitude of the supplier’s marginal cost reduction capability (i.e., it increases $\frac{\partial V(x_1, x_2, \phi)}{\partial \phi}$). In other words, it decreases the marginal capability of the supplier. Finally, A3c states that supplier production effort and capability are complements. Given the product specification, the unique cost minimizing process specification is increasing in the capability of the suppliers. This assumption will play a key role in the design of product-process screening contracts: The more capable suppliers will reveal their capability by choosing a more challenging process.

An important assumption is A4, which states that the prior density $\pi(\phi)$ has a monotone hazard rate; that is, the probability that a supplier’s capability lies in the interval $[\phi, \phi + d\phi]$, given that it is no less than $\phi$, is increasing in $\phi$. As explained in Fudenberg

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$^2$Complements and substitutes have attracted considerable attention in the economics literature lately (see Topkis (1998)), but their role in operations management has largely been unexplored.
and Tirole (1998), many common distributions have this property. Its main implication is that the optimal product and product-process screening contracts will have an appealing monotonicity property – they will screen the more capable suppliers by offering higher information rents in exchange for a choice of a more detailed product specification and a more demanding production process.

3 Model Analysis and Theorems

The analysis proceeds in four steps. First, we consider the single-party scenario where the buyer knows the supplier’s production capability and chooses both product and process specification. This solution presents an unattainable ideal, but provides a benchmark that identifies the distortions that may be caused as a result of decentralization and information asymmetry. Next, we analyze the single product specification contract, followed by the product-process screening contract, and conclude with the product screening contract.

3.1 Single-Party Problem: First-Best

In the single party problem, the buyer knows $\phi$, and chooses $x_1$ and $x_2$ in order to minimize total cost. The problem is as follows: For each $\phi \in \Phi$, find $x_1(\phi)$ and $x_2(\phi)$ to minimize the production cost

$$V(x_1(\phi), x_2(\phi), \phi)$$

subject to

$$0 \leq x_1(\phi) \leq R.$$  \hspace{1cm} (3)

The optimal (first-best) choice of product specification and production effort for each type $\phi \in \Phi$ will be denoted $x_1^*(\phi)$ and $x_2^*(\phi)$, respectively. The main result is summarized below.
Proposition 1  Under assumption A1, the optimal solution to (2)-(3) is:

\[ x_1^*(\phi) = R \]  
\[ V_{x_2}(R, x_2^*(\phi), \phi) = 0 \quad \text{for each } \phi \in \Phi \]  

3.2 Single Product-Specification Contract

In this contract the buyer proposes a single product specification and price. The problem is as follows: The buyer specifies \( x_1 \) and a price \( t \), and the supplier (assuming he accepts the contract), chooses \( x_2(\phi) \) that minimizes his total cost, given the product specification represented by \( x_1 \). Then the buyer’s problem is to determine \( x_1 \in [0, R] \) and \( t \) to

\[
\text{minimize } t \\
\text{subject to the participation constraint that the supplier’s profit will exceed a reservation profit level } u, \\
\max_{x_2} [t - V(x_1, x_2, \phi)] \geq u \quad \text{for each } \phi
\]  

It is easy to see that the buyer will set \( x_1 \) equal to the capacity constraint \( R \), because otherwise, she would forego some cost savings. In response to this product specification, a supplier of type \( \phi \) will choose the process \( x_2 \) that minimizes the production cost \( V(R, x_2, \phi) \). This coincides with first-best \( x_2^*(\phi) \). Hence, the optimal single product-specification contract induces first-best choices by the buyer and the supplier.

Let us now turn to the price \( t \). First, it follows from (7) that the price must ensure the participation of the supplier with the highest cost (i.e. lowest capability \( \phi = \phi \)). Hence, the price is equal to this supplier’s cost; that is,

\[ t = u + V(R, x_2^*(\phi), \phi). \]  

Note that a supplier with a capability \( \phi > \phi \) is left with a profit (surplus) that reflects his cost advantage over the least capable (i.e. highest cost) supplier.
In conclusion, this contract induces first-best choices by the two parties, but the price paid by the buyer must be enough to cover the cost of the least-capable supplier. Therefore, with a single product specification, the buyer cannot exploit any of the cost savings achieved by a capable supplier.

### 3.3 Product-Process Screening Contract

The previous analysis suggests that the buyer must utilize “screening contracts” in order to optimize her total purchasing cost. Screening contracts provide a menu of options and prices to the supplier, and the supplier reveals his capability by the choice he makes. Product-process screening contracts specify both the product and process specification. That is, for a supplier of type $\phi$, the screening contract specifies a product-process pair $x_1(\phi)$ and $x_2(\phi)$, and a price $t(\phi)$. The intention is that a type-$\phi$ supplier will find it to his best interest to choose the pair $(x_1(\phi), x_2(\phi))$.

The problem of obtaining the optimal screening contract is formulated as follows: Determine a menu of product-process pairs $(x_1(\phi), x_2(\phi))_{\phi \in \Phi}$, and a price menu $(t(\phi))_{\phi \in \Phi}$ to minimize

$$\min_{x_1(\phi), x_2(\phi), t(\phi)} \int_{\Phi} \pi(\phi)t(\phi)d\phi,$$

subject to the incentive compatibility constraint that a supplier of type $\phi$ will find it to his best interest to choose the product-process pair $(x_1(\phi), x_2(\phi))$:

$$t(\phi) - V(x_1(\phi), x_2(\phi), \phi) \geq t(\hat{\phi}) - V(x_1(\hat{\phi}), x_2(\hat{\phi}), \phi)$$

for all $\hat{\phi} \in \Phi$, (10)

and the participation constraint

$$t(\phi) - V(x_1(\phi), x_2(\phi), \phi) \geq u_\bar{u}$$

for each $\phi \in \Phi$. (11)

The optimal product-process screening contract will be denoted by the triple $(x_1^{**}(\phi), x_2^{**}(\phi), t^{**}(\phi))_{\phi \in \Phi}$.

The analysis of this mechanism design problem is standard, and described in Fudenberg and Tirole (1991), chapter 7. The analysis proceeds in two main steps. First, for any
given menu of product-process pairs \((x_1(\phi), x_2(\phi))_{\phi \in \Phi}\), find the price menu \((t(\phi))_{\phi \in \Phi}\) that will implement it; i.e., make it desirable for a type-\(\phi\) supplier to choose the pair \((x_1(\phi), x_2(\phi))\).

Second, obtain the menu of product-process pairs that when implemented with the pricing mechanism from step 1 minimizes the buyer’s total expected cost.

The main result for step 1 is given below.

**Proposition 2**

1. Under assumption A3a,c a menu of product-process pairs \((x_1(\phi), x_2(\phi))_{\phi \in \Phi}\), where \(x_1(\phi)\) and \(x_2(\phi)\) are non-decreasing, is implementable. The pricing mechanism that implements this menu is given by

\[
t(\phi) = u + V(x_1(\phi), x_2(\phi), \phi) - \int_{\phi}^{\Phi} V_\phi(x_1(\phi), x_2(\phi), \phi) d\phi
\] (12)

2. Under assumption A3b,c a menu of product-process pairs \((x_1(\phi), x_2(\phi))_{\phi \in \Phi}\) where \(x_1(\phi)\) is non-increasing and \(x_2(\phi)\) is non-decreasing is implementable. The transfer payment that implements this menu is given by (12).

The nature of the non-linear pricing system (12) is insightful. The total price for a supplier of type \(\phi\) is equal to the reservation profit margin, plus the total cost of production when the supplier chooses the desired pair \(x_1(\phi), x_2(\phi)\), plus an information rent that is increasing in the supplier’s type. This makes it unattractive for the supplier to choose any other pair but the pair tailored to his capability.

Let us now turn to the second step of the analysis, which is to identify the menu of product-process pairs that will minimize the buyer’s expected total cost when implemented with the pricing mechanism (12). If we ignore the requirement that this mechanism implements policies that are monotone, then our problem becomes: Find a menu of product-process pairs \((x_1(\phi), x_2(\phi))_{\phi \in \Phi}\) to minimize

\[
\int_{\phi}^{\Phi} \left[ V(x_1(\phi), x_2(\phi), \phi) - \int_{\phi}^{\Phi} V_\phi(x_1(\phi), x_2(\phi), \phi) d\phi \right] \pi(\phi) d\phi;
\] (13)
where (13) is derived by substituting (12) into (9). An application of the integration by parts formula implies that this problem is equivalent to minimizing

$$\int_\Phi [V(x_1(\phi), x_2(\phi), \phi) - \eta(\phi)V_\phi(x_1(\phi), x_2(\phi), \phi)] \pi(\phi) d\phi.$$  \hspace{1cm} (14)

Then, the second step is to obtain the menu \((x_1(\phi), x_2(\phi))_{\phi \in \Phi}\) that minimizes (14), ignoring the monotonicity constraint, and then verify that this constraint is satisfied. This verification is straightforward under assumptions A5-A7.

Before we proceed with the analysis, it is worth noting here that (14) highlights the trade-off that needs to be balanced by the optimal screening contract: The buyer wishes for the supplier to choose the production process that minimizes the total production cost but also wants to extract some of the cost savings. To achieve that she has to offer the supplier an information rent. The information rent that would implement the first-best product-process specifications is excessive. Hence, the menu designed by the buyer proposes a production process and possibly product specification that may be different from first-best. If so, then this increases the direct cost of production compared to first-best but involves lower information rents. In summary, delegation of decision-making authority from the principal to the agent may lead into distortion of product specification and production process in order to induce truthful revelation of the supplier’s capability at a minimum expected cost.

Our main results depend on whether the supplier’s capability and buyer’s product specifications are complements or substitutes. The first result is for the substitutes case.

**Proposition 3** Under assumptions A1, A2, A3b,c, A4-A6 the optimal menu of product-process pairs satisfies the monotonicity condition described in Proposition 1, and, therefore, it can be implemented using the pricing mechanism (12). In the optimal menu, the resource constraint is binding in each \(\phi \in \Phi\), and thus \(x_1^*(\phi) = R\). The optimal process specification...
is given by the solution to the following first-order condition

\[ V_{x_2}(R, x_2^{**}(\phi), \phi) - \eta(\phi)V_{x_2}(R, x_2^{**}(\phi), \phi) = 0 \quad \text{for} \quad \phi \in \Phi. \]  
(15)

where \( x_2^{**}(\phi) \leq x_2^*(\phi) \).

The main insight here is that the buyer will provide the most detailed product specification possible, i.e. \( x_1^{**}(\phi) = R \), regardless of the capability of the supplier. This is because increasing the resources dedicated to product specification has a dual effect: a) it decreases the supplier’s production cost; b) it reduces the information rents paid to the supplier. On the other hand, (15) states that there is a trade-off between process choice and information rents. The optimal process choice is less than first-best in order to balance optimally the twin objectives of minimizing production cost and minimizing information rents.

Let us now turn to the complements case. The following proposition demonstrates that product specifications now have discriminatory power and hence the capacity constraint is not always binding.

**Proposition 4** Under assumptions A1, A2, A3a,c, A4-A7 the optimal menu of product-process pairs satisfies the monotonicity condition described in Proposition 1, and, therefore, it can be implemented using the transfer payment function (12). In addition, there exist two thresholds, \( \phi_L^{**} \) and \( \phi_U^{**} \), with \( \underline{\phi} \leq \phi_L^{**} < \phi_U^{**} \leq \bar{\phi} \), such that

\[
\begin{align*}
x_1^{**}(\phi) &= 0 & \text{for} \ \phi \leq \phi_L^{**} \\
0 &\leq x_1^{**}(\phi) \leq R & \text{for} \ \phi_L^{**} \leq \phi \leq \phi_U^{**} \\
x_1^{**}(\phi) &= R & \text{for} \ \phi_U^{**} \leq \phi.
\end{align*}
\]

The optimal menu of product-process pairs is derived as follows:

\[
\begin{align*}
V_{x_1}(x_1^{**}(\phi), x_2^{**}(\phi), \phi) - \eta(\phi)V_{x_1}(x_1^{**}(\phi), x_2^{**}(\phi), \phi) &= 0 \quad \text{for} \ \phi \in [\phi_L^{**}, \phi_U^{**}] \\
x_1^{**}(\phi) &= 0 \quad \text{for} \ \underline{\phi} \leq \phi \leq \phi_L^{**} \\
x_1^{**}(\phi) &= R \quad \text{for} \ \phi_L^{**} \leq \phi \leq \bar{\phi} \\
V_{x_1}(0, x_2^{**}(\phi_L^{**}), \phi_L^{**}) - \eta(\phi_L^{**})V_{x_1}(0, x_2^{**}(\phi_L^{**}), \phi_L^{**}) &= 0, \\
V_{x_1}(R, x_2^{**}(\phi_U^{**}), \phi_U^{**}) - \eta(\phi_U^{**})V_{x_1}(R, x_2^{**}(\phi_U^{**}), \phi_U^{**}) &= 0
\end{align*}
\]

(17)
\[ V_{x_2}(x_1^{**}(\phi), x_2^{**}(\phi), \phi) - \eta(\phi)V_{x_2\phi}(x_1^{**}(\phi), x_2^{**}(\phi), \phi) = 0 \text{ for } \phi \in \Phi. \quad (18) \]

There now exists a trade-off between resources dedicated to product specification and information rents. While the buyer may wish to utilize all his product specification resources to minimize the total production cost, this induces an increase in the information rents because of the complementarity property A3a. Hence, the buyer finds it desirable to withhold some resources used for product specifications in order to contain the information rents. These resources are used to discriminate suppliers according to their capability as follows: a) suppliers of low capability (i.e. \( \phi \leq \phi_L \)) are bunched together and choose a product specification that involves no effort from the buyer; b) suppliers of high capability (i.e. \( \phi \geq \phi_H \)) are bunched together and choose the product specification that utilizes all buyer resources; and c) suppliers in the middle select a unique product specification that is capability-dependent. Further screening of the suppliers is achieved by offering a menu of production processes.

In terms of comparative statics, one can say that the second-best production process may be distorted either upwards or downwards, but the exact direction of the distortion is ambiguous and is influenced by the cross-partial derivatives \( V_{x_1x_2}(x_1, x_2, \phi) \) and \( V_{\phi x_1x_2}(x_1, x_2, \phi) \).

### 3.4 Product-Specification Screening Contract

Under a product-specification screening contract the buyer offers a menu of product specifications and prices. The process specification is not determined by the contract. Mathematically, the analysis of product-specification screening contracts is analogous to that for their product-process counterparts but with the added constraint that the process minimizes the supplier's direct production cost given his choice of product specification.

The contract-design problem is then formulated as follows: Determine a menu of
product specifications \((x_1(\phi))_{\phi \in \Phi}\), and a pricing menu \((t(\phi))_{\phi \in \Phi}\).

\[
\min_{x_1(\phi), t(\phi)} \int_{\Phi} \pi(\phi) t(\phi) d\phi,
\]

subject to the incentive-compatibility constraint that the type-\(\phi\) supplier will find it to his best interest to choose \(x_1(\phi)\)

\[
t(\phi) - \min_{x_2} V(x_1(\phi), x_2, \phi) \geq t(\hat{\phi}) - \min_{x_2} V(x_1(\hat{\phi}), x_2, \phi) \text{ for all } \hat{\phi} \in \Phi,
\]

and the participation constraint

\[
t(\phi) - \min_{x_2} V(x_1(\phi), x_2(\phi), \phi) \geq u \text{ for each } \phi \in \Phi.
\]

Let \(x_{1***}(\phi)\) denote the product specification that will be chosen by a type-\(\phi\) supplier under the optimal product specification screening contract, \(t_{***}(\phi)\) denote the price offered by the buyer to the supplier for that product specification, and \(x_{2***}(\phi)\) denote the process that minimizes \(V(x_{1***}(\phi), x_2, \phi)\).

Note that the incentive compatibility constraint (20) is similar to its product-process analogue (equation (10)) but the expression \(V(x_1(\phi), x_2, \phi)\) in (10) is replaced by \(\min_{x_2} V(x_1(\phi), x_2, \phi)\).

This enforces the requirement that the process choice in the product specification screening contract is implicit, and represents the supplier’s “best response” to his choice of product specification. The similarity between the incentive compatibility constraints for the product specification screening contract and product-process contracts implies that the analysis of the latter will imitate that of the former but with the cost function \(V(x_1, x_2(\phi), \phi)\) replaced by the induced cost function

\[
\tilde{V}(x_1, \phi) = \min_{x_2} V(x_1, x_2, \phi),
\]

and assumptions A2-A6 replaced by their single-variable analogue; the counterpart of A7 is not needed for the analysis of this contract. Specifically, all assumptions about partial derivatives of \(V\) (either first order or higher) involving \(x_2\) are eliminated. In the remaining
assumptions, all partial derivatives of $V$ with respect to $x_1$ are replaced by partial derivatives \( \tilde{V} \) with respect to $x_1$. For example, A3a is replaced by B3a which states that $\tilde{V}_{x_1}(x_1, \phi) < 0$.

The main result is summarized below.

**Proposition 5** Under assumptions, A1, B2, B3a, B4-B6, the optimal menu of product specifications satisfies the following conditions for $\underline{\phi} \leq \phi^*_L \leq \phi^*_U \leq \bar{\phi}$

$$
x_1^*(\phi) = 0 \quad \text{for } \phi \leq \phi^*_L
$$

$$
0 \leq x_1^*(\phi) \leq R \quad \text{for } \phi^*_L \leq \phi \leq \phi^*_U
$$

$$
x_1^*(\phi) = R \quad \text{for } \phi^*_U \leq \phi
$$

(23)

The optimal product specification $x_1^*(\phi)$ and thresholds $\phi^*_L, \phi^*_U$ are derived as follows:

$$
\begin{align*}
\tilde{V}_{x_1}(x_1^*(\phi), \phi) - \eta(\phi)\tilde{V}_{x_1}(x_1^*(\phi), \phi) &= 0 \quad \text{for } \phi \in [\phi^*_L, \phi^*_U] \\
\tilde{V}_{x_1}(0, \phi^*_L) - \eta(\phi^*_L)\tilde{V}_{x_1}(0, \phi^*_L) &= 0, \\
\tilde{V}_{x_1}(R, \phi^*_U) - \eta(\phi^*_U)\tilde{V}_{x_1}(R, \phi^*_U) &= 0.
\end{align*}
$$

(24)

The process specification $x_2^*(\phi)$ chosen by a type-$\phi$ supplier in response to the optimal screening contract satisfies

$$
V_{x_2}(x_1^*(\phi), x_2^*(\phi), \phi) = 0.
$$

(25)

The pricing system that implements the optimal contract is given by

$$
t^*(\phi) = u + \tilde{V}(x_1^*(\phi), \phi) - \int_\phi^{\bar{\phi}} \phi \tilde{V}_{\phi}(x_1^*(\phi), \phi) d\phi
$$

$$
= u + V(x_1^*(\phi), x_2^*(\phi), \phi) - \int_\phi^{\phi^*_U} V_{\phi}(x_1^*(\phi), x_2^*(\phi), \phi) d\phi.
$$

(26)

Finally, the optimal menu of product specifications coincides with a single product-specification contract either if B3a is replaced by B3b, or if

$$
\tilde{V}_{x_1}(R, \underline{\phi}) - \eta(\underline{\phi})\tilde{V}_{x_1}(R, \underline{\phi}) \leq 0.
$$

(28)

Hence, one would expect to see some bunching of suppliers in the two extremes of the type-region. That is, high-capability suppliers are bunched together, as are low-capability ones. The optimal screening contract uses differentiated product specification to discriminate
between suppliers in the middle. The proposition also identifies, through (28), conditions under which complete bunching of all supplier types occurs; that is, when the optimal product-specification screening contract becomes a single product-specification contract. First, complete bunching occurs if \( \tilde{V}_{x_1\phi}(x, \phi) \geq 0 \) because in this case, increasing \( x_1 \) has the dual effect of reducing direct costs and reducing information rents. Second, complete bunching can also occur even if \( \tilde{V}_{x_1\phi}(x, \phi) \leq 0 \), as long as 
\[
\tilde{V}_{x_1\phi}(R, \phi) - \eta(\phi)\tilde{V}_{x_1\phi}(R, \phi) \leq 0;
\]
because in this case it follows that \( x_{1\ast}(\phi) = R \) and hence \( \phi_{U\ast} = \phi \). However, if A4 (B4) holds (i.e. \( \eta(\phi) = +\infty \)) then complete bunching will not occur since \( \tilde{V}_{x_1\phi}(R, \phi) - \eta(\phi)\tilde{V}_{x_1\phi}(R, \phi) = -\eta(\phi)\tilde{V}_{x_1\phi}(R, \phi) = +\infty \). Therefore, complete bunching is optimal when \( \tilde{V}_{x_1\phi}(x, \phi) \geq 0 \), but it may also be optimal when \( \tilde{V}_{x_1\phi}(x, \phi) \leq 0 \), provided that \( \eta(\phi) < +\infty \).

It is worthwhile to relate this finding to the underlying cost function \( V_{x_1}(x_1, x_2, \phi) \). An application of the envelope theorem (see Mas-Colell, Whinston, and Green, 1995) implies that

\[
\tilde{V}_{x_1\phi}(x, \phi) = V_{x_1\phi}(x, x_2(\phi; x), \phi) - \frac{V_{x_1x_2}(x, x_2(\phi; x), \phi)V_{x_2\phi}(x, x_2(\phi; x), \phi)}{V_{x_2}(x, x_2(\phi; x), \phi)},
\]

where \( x_2(\phi; x) = \arg\min_{x_2} V(x, x_2, \phi) \). One could now use this expression and derive conditions for complete bunching based on the crosspartials of the production cost function. A main insight that emerges from such an analysis is that it is much more likely for complete bunching to be optimal when the buyer’s product effort and the supplier’s capability are substitutes as opposed to complements (this result is also confirmed by our numerical analysis).

We can also derive some simple comparative statics (using the implicit function theorem): If \( V_{x_1x_2} > 0 \), (that is if \( x_1 \) and \( x_2 \) are substitutes) then \( x_{1\ast\ast\ast}(\phi) \leq x_1^*(\phi) \) and \( x_{2\ast\ast\ast}(\phi) \geq x_2^*(\phi) \). If, on the other hand, \( V_{x_1x_2} < 0 \) (that is, \( x_1 \) and \( x_2 \) are complements), then \( x_{1\ast\ast\ast}(\phi) \leq x_1^*(\phi) \) and \( x_{2\ast\ast\ast}(\phi) \leq x_2^*(\phi) \).
4 Numerical Example

The goal of this section is to provide a numerical example that illustrates our main results. We consider a quadratic model
\[ V(x_1, x_2, \phi) = A_1 x_1^2 + A_2 x_2^2 + \frac{1}{1+\phi} \{ B_1 x_1^2 + B_{11} x_1 + B_{12} x_1 x_2 + B_2 x_2^2 + B_{22} x_2 \} \]
and a uniform prior on \( \Phi = [0, 1] \). For brevity, we will use FB, SPSC, PSC and PPSC to refer to the first-best, single product-specification contract, product-screening contract, and product-process screening contract, respectively. We present results for two sets of model parameters that reflect the substitutes and complements case. The results are summarized in Figure 1 which presents the process specification effort, the total cost, and the information rents as functions of the supplier’s capability.

a) Substitutes Case: Let \( A_1 = -1.8, A_2 = -1, B_1 = 0.9, B_2 = 3, B_{12} = -1, B_{22} = -0.5, B_{11} = -1 \). Under first-best, \( x_1^*(\phi) = R \) and \( x_2^*(\phi) = \frac{0.5+R}{1+\phi} \). Note that \( x_2^*(\phi) \) increases with \( \phi \). Integrating across values of \( \phi \), we get that the first-best total expected cost for the buyer is \(-137.28\) when \( R = 10 \). By varying the resource constraint \( R \) one can verify that the total expected cost decreases as \( R \) increases. In addition, the comparative advantage of PPSC compared to SPSC or PSC is small and not very sensitive to \( R \) (results not shown for brevity).

For the SPSC, Figure 1 shows that the product and process specifications coincide with first-best as stated in Proposition 2. More importantly, the PSC is also found to coincide with SPSC not only for the capacity scenario presented in Figure 1, but for every capacity scenario; this follows from equation (28) which, under this numerical example, becomes \(-2.1875 - 0.375R \leq 0\), for all \( R \). The corresponding total cost for the buyer is \( V(R, x^{***}_2(0), 0) = -113.78 \) for \( R = 10 \). As \( R \) increases, the buyer’s total cost decreases (results not shown for brevity). Furthermore, it should be noted that even though the product-process specifications in SPSC and PSC coincide with first-best, the system suffers

---

3While these two scenarios do not satisfy assumptions A3a or A3b, they generate results that are consistent with the results generated by these two assumptions (i.e. Propositions 3 and 4).
from agency losses (i.e. the buyer’s total expected cost exceeds first-best). Finally, under the optimal PPSC, it is again optimal to set \( x_1^{**}(\phi) = R \). However, the corresponding optimal supplier effort is \( x_2^{**}(\phi) = \frac{0.5 + R}{5 - 2\phi - \phi^2} \) which increases with \( \phi \). Moreover, \( x_2^{**}(\phi) \leq x_2^{*}(\phi) \) for all values of \( \phi \) as shown in Proposition 3.

The results are illustrated graphically in Figure 1 for the case \( R=10 \). Panel 1 shows the difference in supplier effort among the three contract mechanisms with the PPSC contract generating the lowest of the three. Moreover, supplier effort is increasing in his capability.

Panel 2 shows that the total cost for the buyer decreases with supplier capability under FB and PPSC, illustrating that these mechanisms take advantage of the cost-reduction capability of the more capable supplier. Panel 3 shows that the supplier’s surplus (defined as the information rent) increases with supplier capability. The least-capable supplier has zero surplus, while more capable suppliers are rewarded more for their elevated effort level.

We also note that the PPSC contract leaves the buyer with the lowest surplus among the three mechanisms. However, the comparative advantage of PPSC relative to PSC and SPSC is practically insignificant.

b) Complements Case: The cost function parameters are identical to these in case (a) with a single exception: \( A_1 = -0.9 \). As above, the first-best product-process specifications are: \( x_1^{*}(\phi) = R \) and \( x_2^{*}(\phi) = \frac{0.5 + R}{4 - 2\phi} \). Similarly, the optimal SPSC is as before: it induces first-best decisions but the price is too high. Let us next turn to PSC and PPSC.

Unlike the substitutes case, in the complements case, both contracts offer a menu of product specifications. Specifically, for PSC we obtain \( \phi_{L}^{***} = 0 \) and \( \phi_{U}^{***} = 0.2808 \) for \( R = 10 \). Thus, \( x_1^{***}(\phi) = \min\{R, \frac{4.86 - 4.72\phi + 1.25\phi^2}{3.17 - 11.44\phi + 4.72\phi^2 + 2\phi^3 - \phi^4}\} \). The value of \( \phi_{L}^{***} \) is zero independently of \( R \), while the value of \( \phi_{U}^{***} \) is increasing in \( R \) (see Table 1). The corresponding values of \( x_2^{***}(\phi) = \frac{0.5 + x_1^{***}(\phi)}{4 - 2\phi} \). Finally, under the PPSC, we again find that \( \phi_{L}^{**} = 0 \) (i.e. it is never optimal to set \( x_1^{**}(\phi) = 0 \)). Moreover, \( \phi_{U}^{**} = 0.2737 \) and \( x_1^{**}(\phi) = \min\{R, \frac{6.11 - 2.22\phi - 1.11\phi^2}{3.89 - 12\phi - 2\phi^3 + 4\phi^4 + \phi^5}\} \). The corresponding supplier effort is \( x_2^{**}(\phi) = \frac{0.5 + x_1^{**}(\phi)}{5 - 2\phi - \phi^2} \). Table 1 summarizes the impact of
Figure 1: Process specification effort, total cost, and agent surplus (information rent) as a function of the buyer’s capability.
Table 1: The impact of resource availability on the buyer’s expected total cost (ETC) and on the transition points $\phi^{**}_U$, $\phi^{***}_U$.

<table>
<thead>
<tr>
<th>$R$</th>
<th>First-Best ETC</th>
<th>SPSC ETC</th>
<th>PSC $\phi^{***}_U$ ETC</th>
<th>PPSC $\phi^{**}_U$ ETC</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-47.28</td>
<td>-23.78</td>
<td>0.28</td>
<td>-29.56</td>
</tr>
<tr>
<td>20</td>
<td>-172.88</td>
<td>-72.53</td>
<td>0.303</td>
<td>-103.12</td>
</tr>
<tr>
<td>30</td>
<td>-376.81</td>
<td>-146.28</td>
<td>0.310</td>
<td>-221.6</td>
</tr>
<tr>
<td>40</td>
<td>-659.08</td>
<td>-245.03</td>
<td>0.314</td>
<td>-385.06</td>
</tr>
<tr>
<td>50</td>
<td>-1019.69</td>
<td>-368.78</td>
<td>0.317</td>
<td>-593.5</td>
</tr>
<tr>
<td>100</td>
<td>-3997.82</td>
<td>-1362.53</td>
<td>0.321</td>
<td>-2310.78</td>
</tr>
</tbody>
</table>

$R$ on the buyer expected total cost under each of the mechanisms. Unlike case (a), the total expected costs not only decrease as $R$ increases, but the comparative advantage of PPSC and PSC relative to SPSC increases substantially with $R$.

The main results are illustrated graphically in Figure 1 for the case $R = 10$. Panel 4 demonstrates the effect of the “transition” points $\phi^{**}_U$ and $\phi^{***}_U$ on the buyer’s process specification effort. While for $\phi$ greater than the transition points, the buyer’s effort “tracks” with his first-best effort, similar tracking does not occur prior to the transition points. In addition, panel 4 shows that the supplier effort under PPSC is lower than other contracts, this is not a general result but the effect of problem parameters. Furthermore, Panels 5 and 6 not only confirm the main insights obtained in the substitutes case, but they also demonstrate that unlike the substitutes case, in the complements case the buyer can extract almost all surplus from the low capability suppliers (i.e. suppliers below the “transition” points). However, the supplier surplus increases beyond these transition points in a fashion that resembles the substitutes case.
5 Model Insights and Observations on Practice

In this section we will highlight the insights the analysis of our model provides for product and process development. We also provide observations on practice that relate to these insights.

Perhaps the most important insight is the importance of the contracting mechanism itself. In particular, if we let “ETC” represent expected total cost, then given any uncertainty about supplier capability, it should be clear that:

\[ \text{ETC}_{SPSC} \geq \text{ETC}_{PSC} \geq \text{ETC}_{PPSC} \geq \text{ETC}_{\text{First Best}}. \]

Correspondingly, letting “SS” represent expected supplier surplus; i.e., the information rent paid to the supplier to induce revelation of his capability, then:

\[ \text{SS}_{SPSC} \geq \text{SS}_{PSC} \geq \text{SS}_{PPSC} \geq \text{SS}_{\text{First Best}} = 0. \]

Of course, the buyer’s ability to infer the supplier’s capability also increases left-to-right in these relationships. Indeed, it is this ability that induces these relationships. However, it should also be noted that it is the nature of the menu(s) of contracts derived from our analysis that induces the supplier to reveal his capability.

With respect to practice, it is important to note that a single product specification contract has the potential for very high total cost unless supplier capability is known. We believe this result sheds light on the Japanese practice of limiting “black-box” designs to only a select few of its suppliers, as described by Kamath and Liker (1994). In addition, the relationships above provide an interpretation of why Japanese manufacturers’ invest the effort they do to improve supplier capability, as described by Liker and Wu (2000) and Dyer and Nebeoka (2000).

Our analysis also demonstrates the importance of complementarity versus substitutability in determining the appropriate amount of effort for the buyer to expend on product specification. In particular, depending on the nature of the relationship - substitute or
complement - and the nature of the contract itself, the optimal level of buyer effort either equals zero; the maximum possible, R; or something strictly in-between.

In practice, we do not know if complementarity and substitutability are both realized, but to the extent that they are, then it is important that the buyer assess the nature of its relationship - complementary or substitute - with suppliers and potential suppliers: First, in order to provide the appropriate level of product specification; and, second; if appropriate, to change the nature of that relationship. Further, if both complementarity and substitutability do exist, then their existence provides one important motivation for manufacturers to “qualify” suppliers and/or offer programs such as those described by Dyer and Nebeoka (2000) to align suppliers or potential suppliers.

The results of our analysis further indicate that supplier capability is important not only because, by assumption, more capable suppliers produce at lower cost for the same level of effort, but, under the menu(s) of contracts we derive, more capable suppliers expend more effort than less capable suppliers. Hence, more capable suppliers have the potential to provide much lower total cost, as our numerical example indicates.

With respect to practice, the notion that more capable suppliers expend more effort at cost reduction is, of course, part of Japanese buyer-supplier folklore. However, our analysis indicates that this behavior may be the self-interested logical consequence of the appropriate “contract” or “menu of contracts” between buyer and supplier. Further, this result may provide yet another reason why sophisticated buyers (e.g., Japanese auto and consumer-electronics manufacturers) have extensive programs to both find more capable suppliers and to improve their capability.

6 Literature Review

We are unaware of any studies that examine “product development” and “production” from our general, normative perspective. Nonetheless, our models and analysis are firmly based
in the broad area of product-process development. For example, our model is related to many of those reviewed by Krishnan and Ulrich (2001). Although their review is restricted to development projects within a single firm, Krishnan and Ulrich’s perspective, like ours, is focused on decision making. Further Krishnan and Ulrich’s four categories of decisions - “concept development”, “supply chain design”, “product design”, and “production ramp-up and launch” - are quite similar to our model’s three steps, if both concept development and supply chain design are viewed as part of concept specification.

An interesting backdrop for our model and its results is provided by Clark’s study (1989) of parts strategy and supplier involvement in product development in the world auto industry provides. Specifically, Clark found systematic differences between Japanese automakers and their US and European counterparts with respect to what we model as buyer effort in product design. In particular, “black-box” parts - “those parts whose functional specification is done by the assemblers (buyers), while detailed engineering is done by parts suppliers” - account for 62% of the Japanese automakers’ total procurement costs, but only 16% and 32% for the US and European automakers, respectively. Correspondingly, “detail-controlled” parts - “those parts that are developed entirely by the assemblers (buyers), from functional specifications to detailed engineering drawings” - account for only 30% of Japanese purchases, compared to 81% and 54% for the US and European automakers. Although the results of our analysis cannot be used to explain why these differences occur, one possible explanation is that there may be differences in the prevalence of complementarity and substitution between Japanese, US, and European manufacturers.

Agency Models in Managing Buyer-Supplier Relationships. To the best of our knowledge, Reyniers and Tapiero were the first to model the buyer-supplier relationship from an operations management perspective. In particular, in (1995a) they model a buyer and supplier as players in a zero-sum game, wherein the supplier can control the effort invested in the delivery of quality and the buyer may or may not inspect incoming materials.
Reyniers and Tapiero (1995b) examines a similar relationship and models the effect of contract parameters such as price rebates and after-sales warranty cost on the choice of quality by the supplier.

Kim (2000) examines a buyer-supplier relationship in which the buyer subsidizes supplier effort to reduce cost, thereby increasing channel profit. Baiman, Fischer, and Rajan (2001) employ an agency model in a buyer-supplier relationship to examine contracting issues with respect to internal and external failures. Their results indicate that in choosing a product architecture (e.g., bill of material), the buyer should consider both manufacturability and its implications for contractibility and efficiency in managing the buyer-supplier supply chain.

Liron (1999) examines a joint product-process development contract in a two-party supply chain similar to our’s and provides sharp insights by considering both a static and a dynamic model. His analysis focuses on the free-rider problem (that is, when one party provides less than the desired input because he knows that the other party will cover for the difference) but it does not consider information asymmetry or contractual limitations. Whang (1992) develops a game-theoretic model in which an outside contractor is hired to develop a software for a buyer. That paper shares some common features with our model. Specifically, like the supplier in our model, the developer of the software in the Whang model has private information about the production cost. However, in the Whang model the buyer cannot influence the supplier’s cost by her actions. In fact, one can view our work as a hybrid that includes features of the Liron model (joint product-process development by two parties) and of the Whang model (better informed supplier). We believe that both features are essential to the problem.

**Agency Models in Supply-Chain (Production and Inventory) Management.**

Cachon (1998) reviews the literature on supply-chain management from an agency perspective. Corbett and Tang (1998) provide a framework for addressing several questions related
to contracting under asymmetric information. More recently, Van Miegham (1999) examines three types of contracts between a manufacturer and a subcontractor in a two-stage, two-market game involving the determination of capacity and production to satisfy stochastic customer demand. Cachon and Lariviere (1999) use an agency model to examine truth-telling mechanisms involving capacity choice and inventory-allocation policies for a single supplier and several buyers (i.e., retailers) facing stochastic demand. Cachon and Zipkin (1999) examine cooperative and competitive inventory policies in a two-stage serial supply chain facing stochastic demand. Corbett (2001) examines (Q,R) inventory management using an agency model.

**Agency Models in Economics.** The analytical underpinnings for our work are provided by the principal-agent paradigm. An introductory description of the basic model is given in Mas-Colell, Whinston, and Green (1995), where its two main variations are described: hidden action (moral hazard) where the agent’s actions are unobservable, and hidden information (adverse selection) where the agent has private information about the difficulty of the task he is to perform on behalf of the principal. The hidden information scenario (which is the one we consider here) generates the so-called mechanism design problem: design a mechanism that will induce the agent to reveal his private information. Chapter 7 of Fudenberg and Tirole (1991) analyzes the mechanism design problem and provides numerous references. A more advanced treatment of the topic is provided in Wilson (1995).

## 7 Concluding Remarks

We have developed a principal-agent model for product and process development. In this model, a buyer (principal) delegates production of a “product” to a supplier (agent) who is privately informed about his capability. The buyer is in charge of product specification, while the supplier selects the process specification. We analyze three contract mechanisms including two screening contracts.
The model takes a very simplistic view of a product-process development project. It assumes that all the decisions are static, that the quality of the product and its demand is fixed, and that the relationship between the two parties is not ongoing. Most of these assumptions are apt to be violated in practice and this creates a contracting situation much richer and complex than the one considered here. The development of a higher-resolution model that would relax some of these assumptions is left as a topic for future research.

Our analysis shows that coordinating the efforts of the buyer and the supplier is challenging and requires careful balancing of direct cost minimization and incentive provision through the payment of information rents. In particular, delegation of decision-making authority from the buyer to the supplier may lead into distortion of product specification and production process in order to induce truthful revelation of the supplier’s capability at a minimum expected cost. The ideal scenario from the buyer’s perspective is when there are few restrictions on the instruments to be utilized in a contract, and when the dual objectives of direct production cost minimization and information rents containment are aligned.

A Appendix: Main Proofs.

Proof of Proposition 1: Suppose that the constraint is not binding for some \( \phi' \). Then, \( x_1(\phi') < R \). But (A1) states that \( \frac{\partial V(x_1, x_2, \phi)}{\partial x_1} < 0 \), hence one can decrease the total cost by increasing \( x_1(\phi') \) until \( x_1(\phi') = R \). Therefore, the resource constraint is binding in each \( \phi \).

Proof of Proposition 2: The implementability of the menu of product-process pairs follows from Theorem 7.3 in Fudenberg and Tirole (1998), page 261. To derive the expression for the price we proceed as follows (the method is described in Fudenberg and Tirole (1998), and is replicated here for completeness). First, define the indirect utility function

\[
V_1(\phi) = \max_{\phi \in \Phi} t(\tilde{\phi}) - V(x_1(\tilde{\phi}), x_2(\tilde{\phi}), \phi).
\]

Then, Theorem 7.3. of Fudenberg and Tirole (1991) implies that the incentive compatibility
constraint can be restated as follows:

\[ \frac{dt(\phi)}{d\phi} - V_{x_2}(x_1(\phi), x_2(\phi), \phi) \frac{dx_1(\phi)}{d\phi} - V_{x_2}(x_1(\phi), x_2(\phi), \phi) \frac{dx_2(\phi)}{d\phi} = 0; \]  

(32)

which is equivalent to the statement \( \phi = \arg \max_{\tilde{\phi} \in \Phi} t(\tilde{\phi}) - V(x_1(\tilde{\phi}), x_2(\tilde{\phi}), \phi) \). Equation (32), together with an application of the envelope theorem (see Mas-Colell, Whinston, and Green (1995), page 964) to (31), implies that:

\[ \frac{dV_1(\phi)}{d\phi} = -V_\phi(x_1(\phi), x_2(\phi), \phi) + \frac{dt(\phi)}{d\phi} - V_{x_1}(x_1(\phi), x_2(\phi), \phi) \frac{dx_1(\phi)}{d\phi} \]

\[ - V_{x_1}(x_1(\phi), x_2(\phi), \phi) \frac{dx_2(\phi)}{d\phi} \]

\[ = -V_\phi(x_1(\phi), x_2(\phi), \phi). \]

(33)

(34)

It follows that

\[ V_1(\phi) = u - \int_\phi \varphi V_\phi(x_1(\phi), x_2(\phi), \phi) d\phi. \]

(35)

Hence,

\[ t(\phi) = u + V(x_1(\phi), x_2(\phi), \phi) - \int_\phi \varphi V_\phi(x_1(\phi), x_2(\phi), \phi) d\phi. \]

(36)

Proof of Proposition 3: We proceed in three steps. First, we show that the resource constraint is binding in each period. Then, derive the first-order conditions and confirm that they are necessary and sufficient for the optimization problem without the monotonicity constraint. Lastly, we confirm that the solution to the first-order condition satisfies the monotonicity constraint.

1°. Suppose that for some \( \phi, x_1^{*\ast}(\phi) < R \). Now,

\[ V_{x_1}(x_1(\phi), x_2(\phi), \phi) - \eta(\phi) V_{x_1}(x_1(\phi), x_2(\phi), \phi) < 0, \]

(37)

which implies that one can increase \( x_1(\phi) \) and reduce total cost. This is a contradiction, hence \( x_1^{*\ast}(\phi) = R \) for all \( \phi \in \Phi \).
Therefore, \( \phi \). Straightforward algebra shows that the first-order condition for \( x_2(\phi) \) is (15). To confirm that this condition is necessary and sufficient, consider the second order condition:

\[
V_{x_2x_2}(R, x_2(\phi), \phi) - \eta(\phi)V_{x_2x_2}(R, x_2(\phi), \phi) \geq 0, \tag{38}
\]

because \( V_{x_2x_2}(R, x_2(\phi), \phi) \geq 0 \) (assumption A1), and \( V_{x_2x_2}(R, x_2(\phi), \phi) \leq 0 \) (assumption A6).

3. Finally, we need to confirm the monotonicity of \( x_2^*(\phi) \). To do that, we need to totally differentiate (15) with respect to \( \phi \):

\[
[V_{x_2x_2}(R, x_2^*(\phi), \phi) - \eta(\phi)V_{x_2x_2}(R, x_2^*(\phi), \phi)] \frac{dx_2^*(\phi)}{d\phi} = V_{x_2x_2}(R, x_2^*(\phi), \phi) \left[ \frac{d\eta(\phi)}{d\phi} - 1 \right] + \eta(\phi)V_{x_2x_2}(R, x_2^*(\phi), \phi). \tag{39}
\]

This implies that

\[
\frac{dx_2^*(\phi)}{d\phi} = \frac{V_{x_2x_2}(R, x_2^*(\phi), \phi) \left[ \frac{d\eta(\phi)}{d\phi} - 1 \right] + \eta(\phi)V_{x_2x_2}(R, x_2^*(\phi), \phi)}{V_{x_2x_2}(R, x_2^*(\phi), \phi) - \eta(\phi)V_{x_2x_2}(R, x_2^*(\phi), \phi)} \geq 0;
\]

because \( V_{x_2x_2}(R, x_2^*(\phi), \phi) \left[ \frac{d\eta(\phi)}{d\phi} - 1 \right] + \eta(\phi)V_{x_2x_2}(R, x_2^*(\phi), \phi) \geq 0 \) (assumptions (A3c), (A4), (A5)), and \( V_{x_2x_2}(R, x_2^*(\phi), \phi) - \eta(\phi)V_{x_2x_2}(R, x_2^*(\phi), \phi) \geq 0 \) (assumptions (A1), (A6)). Therefore, \( x_2^*(\phi) \) is increasing in \( \phi \).

Proof of Proposition 4: As in Proposition 3, we start by taking the partial derivatives of (9) with respect to \( x_1(\phi) \) and \( x_2(\phi) \):

\[
V_{x_1}(x_1(\phi), x_2(\phi), \phi) - \eta(\phi)V_{x_1x_1}(x_1(\phi), x_2(\phi), \phi), \tag{40}
\]

\[
V_{x_2}(x_1(\phi), x_2(\phi), \phi) - \eta(\phi)V_{x_2x_2}(x_1(\phi), x_2(\phi), \phi). \tag{41}
\]

Assumptions A3a and A4 imply that the partial derivative 40 is positive for all \( x_1 \) and \( x_2 \) when \( \phi = \bar{\phi} \). Hence, it follows by the continuity of \( V_{x_1}(x_1(\phi), x_2(\phi), \phi) - \eta(\phi)V_{x_1x_1}(x_1(\phi), x_2(\phi), \phi) \) that \( x_1^*(\phi) = 0 \) in the neighborhood of \( \phi = \bar{\phi} \). Similarly, assumptions A3a and A4 imply that the partial derivative 41 is negative for all \( x_1 \) and \( x_2 \) when \( \phi = \bar{\phi} \). Hence, the same argument as before shows that \( x_1^*(\phi) = R \) in the neighborhood of \( \phi = \bar{\phi} \). This confirms (16).
Now, we need to verify that for $\phi \in [\phi_L, \phi_U]$.

\[
V_{x_1}(x_1(\phi), x_2(\phi), \phi) - \eta(\phi)V_{\phi x_1}(x_1(\phi), x_2(\phi), \phi) = 0, \tag{42}
\]
\[
V_{x_2}(x_1(\phi), x_2(\phi), \phi) - \eta(\phi)V_{\phi x_2}(x_1(\phi), x_2(\phi), \phi) = 0. \tag{43}
\]

First, by the prior argument it follows that there is a region in which (42)-(43). To complete the proof we need to verify that this region is unique and contiguous and that the first-order conditions are necessary and sufficient. Assumption (A6)-(A7) imply that the Hessian for the function $V(x_1, x_2, \phi) - \eta(\phi)V(x_1, x_2, \phi)$ is diagonally strictly dominant; thus, the Hessian is positive definite. Hence, the first order conditions are necessary and sufficient. Next, if we verify that the solution to (42)-(43) is monotone in $\phi$, it follows that the region in which these conditions hold is contiguous, and the proof is complete.

The proof proceed in two steps. In the regions where $x_1^*(\phi) = 0$ or $x_2^*(\phi) = R$, one can prove that $x_2^*(\phi)$ is non-decreasing in $\phi$ by replicating the arguments in step 3 of the proof of proposition 3. In the region, $\phi \in [\phi_L, \phi_U]$, we proceed as follows: First, totally differentiate (42)-(43) with respect to $\phi$:

\[
[V_{x_2x_2}(x_1^*(\phi), x_2^*(\phi), \phi) - \eta(\phi)V_{x_2x_2\phi}(x_1^*(\phi), x_2^*(\phi), \phi)] \frac{dx_2^*(\phi)}{d\phi} = \frac{\eta(\phi)V_{x_2\phi}(x_1^*(\phi), x_2^*(\phi), \phi) - \eta(\phi)V_{\phi x_2}(x_1^*(\phi), x_2^*(\phi), \phi)}{dx_1^*(\phi)} \tag{44}
\]
and

\[
[V_{x_1x_1}(x_1^*(\phi), x_2^*(\phi), \phi) - \eta(\phi)V_{x_1x_1\phi}(x_1^*(\phi), x_2^*(\phi), \phi)] \frac{dx_1^*(\phi)}{d\phi} = \frac{\eta(\phi)V_{x_1\phi}(x_1^*(\phi), x_2^*(\phi), \phi) - \eta(\phi)V_{\phi x_1}(x_1^*(\phi), x_2^*(\phi), \phi)}{dx_2^*(\phi)} \tag{45}
\]

If we solve equations (45) and (44) with respect to $\frac{dx_1^*(\phi)}{d\phi}$ and $\frac{dx_2^*(\phi)}{d\phi}$ and use assumptions
(A1), (A4), (A5), (A6), (A7), we conclude that $\frac{dz_1^*(\phi)}{d\phi}$ and $\frac{dz_2^*(\phi)}{d\phi}$ are non-negative. The tedious details are omitted for brevity.

**Proof of Proposition 5:** Imitates the proof for Proposition 4 and is omitted.

**References.**


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